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SINGLE INTERFACE GROWTH: FLUCTUATIONS AND THE CORRELATION LENGTH

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Growth behavior of interfaces is usually described by a power-law of the growth in time of the interface width. This general scaling picture is an average behavior description, which may not be valid when only a finite number of interfaces is considered. In this work we study theoretically and experimentally the growth behavior of single interfaces and show that the growth of the interface width always exhibits a non-monotonic, fluctuating behavior. We study numerically the Quenched-noise Kardar-Parisi-Zhang (QKPZ) equation, using different noise distributions, and show that this behavior results from competing mechanisms of normal growth and surface tension forces in this equation. We define a new measure of the interface width fluctuations and present a way to extract the correlation length of the interface from these fluctuations.

Keywords: Interface dynamics; QKPZ equation; fluctuations; correlation length.

1. Introduction

The dynamics and geometry of surfaces and interfaces were extensively studied using the concepts of self-affine and fractal scaling [0-0]. A possible theoretical treatment of a propagating self-affine interface is based on constructing a continuum differential equation for describing the motion of the interface. The simplest nonlinear Langevin equation for a local growth of the profile is given by the Kardar–Parisi–Zhang (KPZ) [0] equation:

$$\frac{\partial h}{\partial t} = F + \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(x,t) \tag{1}$$

where $h(x,t)$ is the interface height at position x at time t, F is a constant force, ν is the surface tension, λ is proportional to the velocity normal to the interface and η is a noise term. A possible variant of the KPZ equation is QKPZ (Quenched noise KPZ) [0–0], in which the noise term depends on the spatial coordinates rather than the time, i.e. $\eta(x,h)$ instead of $\eta(x,t)$. Analysis of these equations under certain conditions can predict sets of scaling exponents [0]. In particular, the width W of the interface, which is formally defined as

$$W^2(L,t) = \langle h(x,t)^2 \rangle - \langle h(x,t) \rangle^2 , \tag{2}$$

has been assumed to obey a power-law in time t and distance L, with two scaling exponents, α, the roughness exponent and β, the growth exponent, defined as

$$W \sim \begin{cases} t^{\beta} & t << t_0 \\ L^{\alpha} & t >> t_0 \end{cases} \tag{3}$$

where $t_0 \approx L^{\alpha/\beta}$, and L is a window varying from the smallest length scale (say a single lattice unit) to the system size L_0 [6].

In this letter we show, theoretically and experimentally, that when a finite number of interfaces is involved, and especially in the case of a single interface, the interface width *W(t)* exhibits fluctuations around the average scaling law. This unique feature of the dynamical growth results from the competition between the mechanisms that appear in the QKPZ equation, and contains hidden information about the system properties. We define a measure for the fluctuations size and show how to extract the correlation length of the interface from this measure.

2. Experimental Data

Our experimental data has been obtained from the interface growth in the spreading process of Hg droplets (150μm in diameter) on thin metallic films (silver – thickness 2000-4000Å or 0.1mm, gold – thickness 1500Å) [0–0]. Such spreading processes are of great interest and importance in material science and technology [0–0].

In Fig. 1 we show the results of log *W(t)* versus log *t* for spreading on a 0.1mm silver foil. The width is measured as a function of time for 8 different window sizes *L*, ranging from 3μm to 25μm. The growth exponent $\beta = 0.60$ is the slope (in time) of each of these curves. As can be seen, β is independent of the window size *L*, but the fluctuations are evident and cannot be ignored. Moreover, since the fluctuations occurrence times are also independent on *L,* we conclude that the fluctuations reflect an intrinsic property of this system.

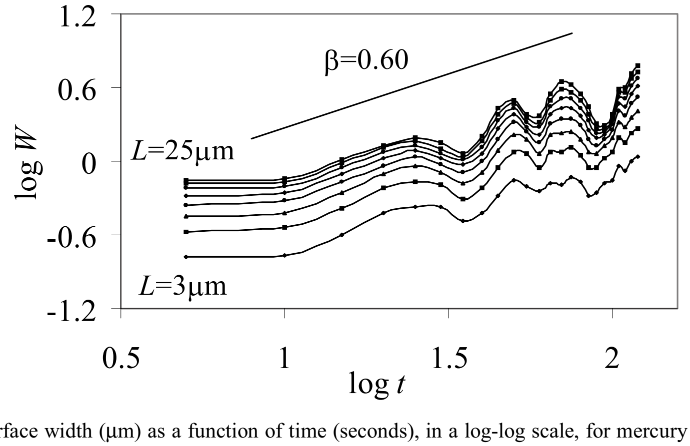

Fig. 1. The interface width (μm) as a function of time (seconds), in a log-log scale, for mercury droplet spread on a 0.1 mm silver foil, for several window sizes *L*, varying from 3 to 25μm.

3. Numerical Data

In order to study these fluctuations and the system properties which they reflect, we solved the QKPZ equation numerically, for a wide range of values for the parameters λ and ν. This version of the KPZ equation resembles the experimental system of Ref. [0] in the sense that surface film defects induce time-independent noise into the system. In the experimental system described above, the metal surface is not perfectly smooth. An Atomic Force Microscope (AFM) scan revealed surface pinning, with a Gaussian distribution of the pins' heights. Therefore, we studied Eq. (4) with *Gaussian noise*, with zero mean and standard deviation of 1. We also studied the case of uniformly distributed noise, in the range (-1,1).

Our numerical system consisted of a 1+1 dimensional interface of 500 points (pixels), initially flat. Each point in the lattice was initialized with a random noise value, according to one of the distributions described above. We used the Forward Time Centered Space (FTCS) scheme [0] for the numerical integration, namely:

$$\frac{\Delta h}{\Delta t} = F + \nu \frac{h(x+\Delta x) - 2h(x) + h(x-\Delta x)}{\Delta x^2} + \frac{\lambda}{2}\left(\frac{h(x+\Delta x) - h(x-\Delta x)}{2\Delta x}\right)^2 + \eta(x,h) \ . \quad (4)$$

F is a constant drifting force, equal to 1, which describes the constant flow [0–0]. We used a space interval $\Delta x = 1$ (discrete lattice) and time intervals of $\Delta t = 0.1$ and $\Delta t = 0.05$. In Fig. 2 we show the results for the width W as a function of time t, for a single interface with λ=0.1, ν=0.15 and uniformly distributed quenched noise. The fluctuations resemble the fluctuations in the corresponding experimental plot in Fig. 1. From the existence of such fluctuations in the relatively simple case of the QKPZ equation, we infer that the fluctuating behavior of the width *W(t)* is a fundamental feature of the interface dynamics, resulting from the different mechanisms in the growth process, i.e. the nonlinear growth (represented by λ) and the surface tension (represented by ν). Very similar growth fluctuations were also obtained in the case of the Gaussian noise.

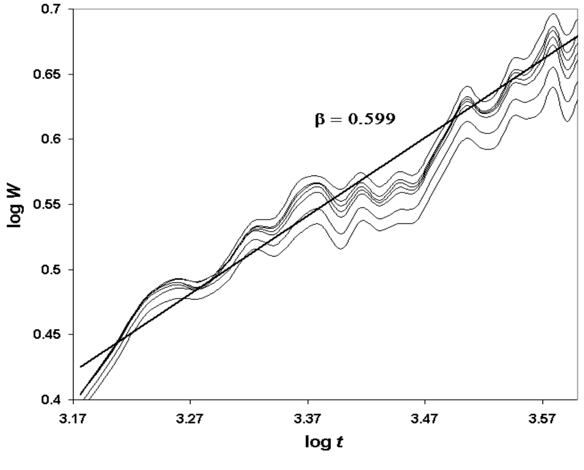

Fig. 2. The width W of the interface as a function of time, in a log-log scale, obtained from the numerical solution of the QKPZ equation for a single interface, for window sizes L = 70, 80, 100, 120, 150, 200, 230 pixels, represented by the different lines (from bottom to top). In this system λ = 0.1, ν = 0.15 and the noise is uniform.

When a local gradient in the interface height occurs, due to the noise, a hump in the interface is created. Then the nonlinear term tends to increase its size whereas the surface tension tends to decrease it. The result of this competition is the fluctuations of the local growth, which are reflected in the overall behavior of the width W as a function of time. It should be noted that the surface tension mechanism is acting in a "time delay" relative to the nonlinear growth, because it responses to the growth *after* it has happened. The advance due to the nonlinear term, and then the regression due to the surface tension, are the reason for the non-monotonic behavior.

The specific shape of the graph of log $W(t)$ versus log t varies from one realization to another. This poses the assumption that within these fluctuations there is a hidden information about the specific system characteristics. In order to extract this information, an estimate for the size of the fluctuations is needed. We calculated the linear fit for the non-monotonic graph of log $W(t)$ versus log t and the deviations from the straight line. We averaged the square of these deviations over the time series and obtained a characteristic value ϕ, defined as

$$\phi = \sqrt{\left\langle (\log W_i - \log W_i^{linear})^2 \right\rangle_i} \tag{5}$$

where W_i is the width of the interface at time i and log $W_i^{\ linear}$ is the value of the linear fit for time i. The brackets denote average over time.

The size of fluctuations, as measured by ϕ, depends on the system parameters ν and λ, which represent the mechanisms of surface tension and normal growth in the KPZ equation (Eq. 1). In Fig. 3 we show the fluctuations size as a function of the ratio ν/λ, for both uniform and Gaussian noise. For both distributions, the measure ϕ is maximal for $\nu/\lambda \sim 1$, namely when the two competing mechanisms are around the same order of magnitude. This supports our previous conjecture that the width fluctuations arise due to this competition.

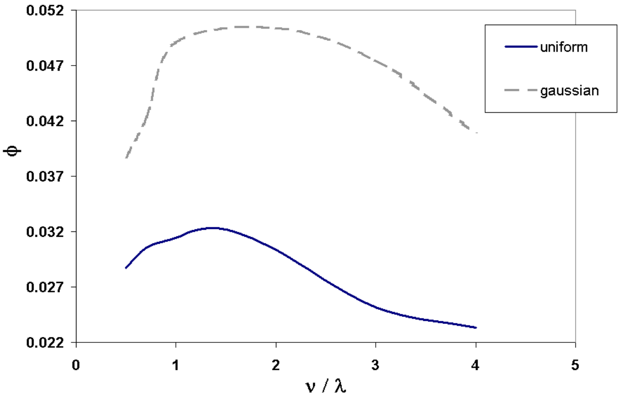

Fig. 3. The fluctuations size ϕ versus the ratio ν/λ, for a system with uniformly distributed noise (solid line) and Gaussian noise (dashed line).

4. Extracting the Correlation Length

In Fig. 4 we show a graph of the fluctuations measure ϕ versus the window length L, for a random system with uniformly distributed noise (λ=0.1 and ν=0.1). We can see that

ϕ grows monotonically with L, up to a certain crossover length L^*=160 pixels. The monotonic growth regime is intuitively clear: when a small window is taken, large changes in the interface cannot be traced, and the fluctuation size is small. When the window is enlarged, a larger section of the interface is analyzed and the average fluctuation size is larger. Above the crossover length L^* the fluctuations measure does not increase anymore. This means that this is a characteristic length, which has the physical features of the entire interface. This is analogous to the definition of the correlation length $\xi_{||}$ [0], which is usually related to the roughness exponent [0]. In Fig. 5 we show the corresponding graph for the roughness exponent (W versus L in a log-log scale) at the end of the process [0–0]. The crossover in the slope occurs at $\xi_{||} = 158$ pixels, which is considered as the parallel correlation length. This length is consistent with the length L^*=160 pixels that we obtained using the fluctuations analysis.

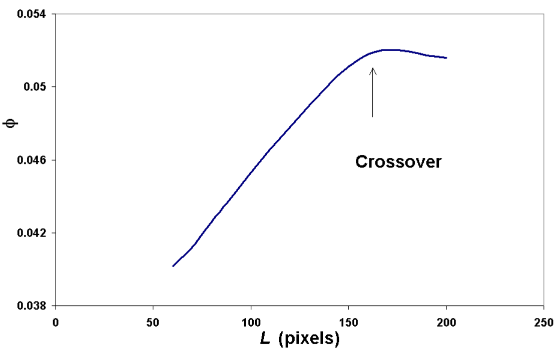

Fig. 4. The fluctuations measure ϕ versus the window length L, for $\lambda = 0.1$, $\nu = 0.1$ and uniformly distributed noise.

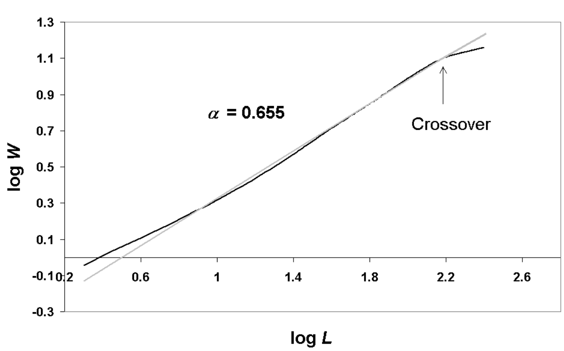

Fig. 5. The width W as a function of L (log-log scale) for the system of Fig. 4. The crossover in the slope (α) reveals the characteristic correlation length, which is 158 pixels.

5. Discussion

For measuring the fluctuations size in the experimental systems, we defined an equivalent method for estimating the size of the fluctuations. We measured the peak-to-peak deviation, by averaging the difference in log$W(t)$ from each maximum point to the following minimum. This measure is labeled as "$\Delta \log W$" [0]. The two measures, ϕ and $\Delta \log W$, differ by their numerical values, but exhibit the crossover at the same L^*. Therefore, they can be evenly used. We measured the fluctuations size for various spreading processes, on thin silver thin films, silver foils and gold films. For example, for the system presented in Fig. 1 we obtained a crossover of $\Delta \log W$ at L^*=8.1µm. The roughness exponent analysis at the end of the process yields a crossover at $L=\xi_{||}$=8.1µm. We see that in both methods (i.e., using the fluctuations measure or the roughness exponent crossover), the obtained characteristic length is consistent. Therefore, we can extract the characteristic correlation length by the interface fluctuations at a much earlier stage of the process.

6. Summary

To summarize, we have shown that interface growth is a non-monotonic dynamical process, and the interface width exhibits noticeable fluctuations around the average scaling law. This phenomenon is observed only when a small number of interfaces is considered. We have shown that the fluctuations' size contains hidden information about the specific systems and developed a method to extract the characteristic correlation length from these fluctuations. Our method was successfully applied to a variety of experimental systems, with different length scales (from µm to cm), as described in detail in [12].

Acknowledgment

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